

Hydrogen in Electrodynamics. IX. The Equations of Second Order

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The equation of second order of electrodynamic hydrogen are derived.

Again we start with covariant source-free electrodynamics [1] (22) with its complex components

$$\left\{ \begin{array}{l} \text{rot } \mathbf{E} + \frac{\mu}{c} \dot{\mathbf{H}} = 0 \\ \text{rot } \mathbf{H} - \frac{\varepsilon}{c} \dot{\mathbf{E}} = 0 \\ \text{div } \varepsilon \mathbf{E} = 0 \\ \text{div } \mu \mathbf{H} = 0 \end{array} \right\} \equiv \left\{ \begin{array}{l} \text{rot } \mathbf{E} + \frac{\mu}{c} \dot{\mathbf{H}} = 0 \\ \text{rot } \mathbf{H} - \frac{\varepsilon}{c} \dot{\mathbf{E}} = 0 \end{array} \right\} \begin{array}{l} \text{div } \mathbf{E} = \text{div } \mathbf{H} = 0 \\ \mathbf{E} \perp \text{grad } \varepsilon \\ \mathbf{H} \perp \text{grad } \mu \end{array}$$

where we have ensured the absence of sources by the requirement

$$\text{div } \mathbf{E} = \text{div } \mathbf{H} = 0. \quad (2)$$

Considering, on the other hand, the well-known differential equations of second order for inhomogeneous media, i.e.,

$$\left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} - \text{grad div} - \frac{1}{\mu} \text{grad } \mu \times \text{rot} \right) \mathbf{E} = 0, \quad (3)$$

$$\left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} - \text{grad div} - \frac{1}{\varepsilon} \text{grad } \varepsilon \times \text{rot} \right) \mathbf{H} = 0, \quad (4)$$

and inserting here (2) too, we get for the system (1) the following equations of second order

$$\left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{E} + \frac{1}{c} \text{grad } \mu \times \dot{\mathbf{H}} = 0, \quad (5)$$

$$\left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{H} - \frac{1}{c} \text{grad } \varepsilon \times \dot{\mathbf{E}} = 0, \quad (6)$$

if we replace the rotors or curls by their equivalents according to (1). Multiplication of (5) by $\dot{\mathbf{E}}$ and of (6) by $-\dot{\mathbf{H}}$ yields

$$\dot{\mathbf{E}} \left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{E} + \frac{1}{c} \text{grad } \mu \times \dot{\mathbf{H}} \cdot \dot{\mathbf{E}} = 0, \quad (7)$$

$$-\dot{\mathbf{H}} \left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{H} - \frac{1}{c} \text{grad } \varepsilon \times \dot{\mathbf{H}} \cdot \dot{\mathbf{E}} = 0, \quad (8)$$

Because of [2] (5)

$$\frac{1}{c} \text{grad } \mu \times \dot{\mathbf{H}} \cdot \dot{\mathbf{E}} = \frac{1}{c} \text{grad } \varepsilon \times \dot{\mathbf{H}} \cdot \dot{\mathbf{E}}, \quad (9)$$

the addition of (7) and (8) gives

$$\dot{\mathbf{E}} \left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{E} = \dot{\mathbf{H}} \left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{H}, \quad (10)$$

from which follows, because of $\mathbf{E} \neq \mathbf{H}$ ([2]),

$$\left(\Delta - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{Q} = 0, \quad (\mathbf{Q} = \mathbf{E}, \mathbf{H}). \quad (11)$$

These are the electrodynamic hydrogen equations of second order, if we insert the hydrogen interfraction [2] (5) for ε and μ .

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[1] H. Sallhofer, Z. Naturforsch. **43a**, 1039 (1988).

[2] H. Sallhofer, Z. Naturforsch. **45a**, 1361 (1990).

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