Hydrogen in Electrodynamics. IX. The Equations of Second Order

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The equation of second order of electrodynamic hydrogen are derived.

Again we start with covariant source-free electrodynamics [1] (22) with its complex components

$$\left.\begin{aligned} \operatorname{rot} \boldsymbol{E} + \frac{\mu}{c}\dot{\boldsymbol{H}} &= 0 \\ \operatorname{rot} \boldsymbol{H} - \frac{\varepsilon}{c}\dot{\boldsymbol{E}} &= 0 \\ \operatorname{div} \varepsilon \boldsymbol{E} &= 0 \\ \operatorname{div} \mu \boldsymbol{H} &= 0 \end{aligned}\right\}_{\operatorname{div}\boldsymbol{E} = \operatorname{div}\boldsymbol{H} = 0} \equiv \left\{\begin{aligned} \operatorname{rot} \boldsymbol{E} + \frac{\mu}{c}\dot{\boldsymbol{H}} &= 0 \\ \operatorname{rot} \boldsymbol{H} - \frac{\varepsilon}{c}\dot{\boldsymbol{E}} &= 0 \end{aligned}\right\}_{\substack{\operatorname{div}\boldsymbol{E} = \operatorname{div}\boldsymbol{H} = 0 \\ \boldsymbol{E} \perp \operatorname{grad}\varepsilon \\ \boldsymbol{H} \perp \operatorname{grad}\mu}}$$

where we have ensured the absence of sources by the requirement

$$\operatorname{div} \boldsymbol{E} = \operatorname{div} \boldsymbol{H} = 0\,. \tag{2}$$

Considering, on the other hand, the well-known differential equations of second order for inhomogeneous media, i.e.,

$$\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2} - \operatorname{grad}\operatorname{div} - \frac{1}{\mu}\operatorname{grad}\mu \times \operatorname{rot}\right)\boldsymbol{E} = 0, \tag{3}$$

$$\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2} - \operatorname{grad}\operatorname{div} - \frac{1}{\varepsilon}\operatorname{grad}\varepsilon \times \operatorname{rot}\right)\boldsymbol{H} = 0, \tag{4}$$

and inserting here (2) too, we get for the system (1) the following equations of second order

$$\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} + \frac{1}{c}\operatorname{grad}\mu \times \dot{\boldsymbol{H}} = 0\,, \tag{5}$$

$$\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{H} - \frac{1}{c}\operatorname{grad}\varepsilon \times \dot{\boldsymbol{E}} = 0\,, \tag{6}$$

if we replace the rotors or curls by their equivalents according to (1). Multiplication of (5) by $\dot{\boldsymbol{E}}$ and of (6) by $-\dot{\boldsymbol{H}}$ yields

$$\dot{\boldsymbol{E}}\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} + \frac{1}{c}\operatorname{grad}\mu \times \dot{\boldsymbol{H}} \cdot \dot{\boldsymbol{E}} = 0\,, \tag{7}$$

$$-\dot{\boldsymbol{H}}\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{H} - \frac{1}{c}\operatorname{grad}\varepsilon \times \dot{\boldsymbol{H}} \cdot \dot{\boldsymbol{E}} = 0\,, \tag{8}$$

Because of [2] (5)

$$\frac{1}{c}\operatorname{grad}\mu \times \dot{\boldsymbol{H}} \cdot \dot{\boldsymbol{E}} = \frac{1}{c}\operatorname{grad}\varepsilon \times \dot{\boldsymbol{H}} \cdot \dot{\boldsymbol{E}}\,, \tag{9}$$

the addition of (7) and (8) gives

$$\dot{\boldsymbol{E}}\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} = \dot{\boldsymbol{H}}\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{H}\,, \tag{10}$$

from which follows, because of $E \neq H$ ([2]),

$$\left(\Delta - \frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)Q = 0\,, \quad (Q = E, H)\,. \tag{11}$$

These are the electrodynamic hydrogen equations of second order, if we insert the hydrogen interfraction [2] (5) for ε and μ.

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[1] H. Sallhofer, Z. Naturforsch. **43a,** 1039 (1988).
[2] H. Sallhofer, Z. Naturforsch. **45a,** 1361 (1990).

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